

Neutron-Deuteron System and Photon Polarization Parameter at Thermal Neutron Energies

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Abstract

Effective Field Theory(EFT) is, the unique, model independent and systematic low-energy version of QCD for processes involving momenta below the pion mass. A low-energy photo-nuclear observable in three-body systems, photon polarization parameter at thermal neutron energies is calculated by using pionless EFT up to next-to-next to leading order(N²LO). In order to make a comparative study of this model, we compared our results for photon polarization parameter with the realistic Argonne v_{18} two-nucleon and Urbana IX or Tucson-Melbourne three-nucleon interactions. Three-body currents give small but significant contributions to some of the observables in the neutron-deuteron radiative capture cross section at thermal neutron energies. In this formalism the three-nucleon forces are needed up to N²LO for cut-off independent results. Our result converges order by order in low energy expansion and also cut-off independent at this order.

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I. INTRODUCTION

The study of the three-body nuclear physics involving nucleon-deuteron photodisintegration of ^3He and ^3H as well as the time reversed nucleon radiative capture by deuteron has been investigated in theoretical and experimental works over the past decays. The photon polarization in the reaction neutron-deuteron capture reaction has been measured with polarized thermal neutrons by Konijnenberg et al. [1]. In addition the search for three-nucleon force effects in the electromagnetically induced process, come more and more into the focus in the recent years. In principle, using the continuity equation for three-nucleon forces lead to three-nucleon currents. It is a quantitative question based on current choices of nuclear force models to reveal signatures by switching on and off three-nucleon forces.

Several groups are studying electromagnetic processes in the three-nucleon system. In Refs. [2, 3], the nucleons are taken as interacting via two- and three-nucleon potentials and the electromagnetic currents are then constructed using the exchange scheme to satisfy the current conservation relation (CCR), but only with a part of the interaction. In another work, the meson exchange currents are taken into account using the Siegert's theorem [4] without three-body currents. Three-body currents are added using a nuclear model which allows for the excitation of a nucleon to a Δ isobar. The Δ excitation yields also effective three-body forces and three-body currents. However, this current model does not satisfy exactly the CCR with the adopted Hamiltonian [5]. In alternative descriptions of the three-nucleon electromagnetic processes for very low energy pd and nd radiative capture, in model dependent theory, a variety of electromagnetic observables involving the two- and three-nucleon forces have been extensively studied in the past by several research groups (for a review, see Ref. [6]). Recently, Viviani et al. have investigated the nd radiative capture reactions below deuteron breakup threshold [7]. Their work shows sensitivity to short-range physics namely, details of including the physics of the Delta and pion-exchange currents. They obtained the cross section from Argonne v_{14} two-nucleon and Urbana VIII three-nucleon interactions (AV14/UVIII), also from Argonne v_{18} two-nucleon and Urbana IX three-nucleon interactions (AV18/UIX), including Δ admixtures. They found Cross section of 0.600 (mb) and 0.578 (mb) which are above the experimental values by 18% and 14%, respectively. It is worthy to mention that the explicit inclusion of Δ -isobar degrees of freedom in the nuclear wave function improve the agreement with the experimental data better than those obtained using the perturbation theory, Δ_{PT} . This indicate that their results for very-low energy observables are sensitive to the details of the short-range part of the interaction. Recent calculations using gauge-invariant currents reduced the spread [8], however including three-body currents results 0.558 mb, which still is above the data by 10%. Model-dependent currents associated with $\Delta(1232)$ were identified as a source of the discrepancy. Thus, the question that remains is, how such details of short-range physics can so severely influence a very long range reaction with maximal energies of less than 10 MeV.

Recently developed pionless EFT is particularly suited for high order precision calculation. The so-called pionless EFT in nuclear physics aspires a systematic classification of all forces. At its heart lies the tenet that physics at those very low energies can be described by point-like interactions between nucleons only. In this approach, all particles but the nucleon themselves are considered high energy degrees of freedom and are consequently “integrated out”. The resulting EFT is considerably simpler than potential models or the “pionful” version of nuclear EFT (in which pions are kept as explicit degrees of freedom), but its range of validity is reduced to typical momenta below the pion mass. There are many processes situated at thermal energies which are both interesting in their own right

and important for astrophysical applications. Recently we have calculated the cross section of radiative capture process $nd \rightarrow {}^3H\gamma$ by using pionless EFT [9, 10]. No new three-nucleon forces are needed up to N²LO in order to achieve cut-off independent results, in addition to those fixed by the triton binding energy and nd scattering length in the triton channel. The cross-section is determined to be $\sigma_{tot} = [0.503 \pm 0.003]mb$.

The present study investigates a low-energy photo-nuclear observable in three-body systems namely, photon polarization parameter at thermal neutron energies, using pionless EFT up to N²LO. The emphasis is on constructing three-body currents with model independent theory corresponding to three-nucleon interactions and comparison of the our model's result with those of other model dependent theory.

The paper is organized as follows: In Section II, we briefly review theoretical framework including the Faddeev integral equation, three-body forces and cut-offs dependence for calculating of neutron-deuteron radiative capture at thermal energies. Then we calculate the photon polarization parameter at thermal energies in Section III and compare our results with the corresponding experimental. Section IV is devoted to comparison of our result with the data and the results of other theoretical models. Summary and conclusions are given in Section V.

II. NEUTRON-DEUTERON RADIATIVE CAPTURE AT THERMAL ENERGIES

At thermal energies the nd capture reaction proceeds through S -wave capture predominantly via magnetic dipole transitions from the initial doublet $J=1/2$ and quartet $J=3/2$ nd scattering states. In addition, there is a small contribution due to an electric quadrupole transition from the initial quartet state. Consequently, ${}^2S_{1/2}$ describes the preferred mode for $nd \rightarrow {}^3H\gamma$ and $pd \rightarrow {}^3He\gamma$. The three-nucleon Lagrangian is well-known and will not be discussed here [14, 15].

As long-distance phenomena must be insensitive to details of the short range physics (and in particular of the regulator chosen), Bedaque et al. [14, 15] showed that the system must be stabilized by a three-body forces

$$\mathcal{H}(E; \Lambda) = \frac{2}{\Lambda^2} \sum_{n=0}^{\infty} H_{2n}(\Lambda) \left(\frac{ME + \gamma_t^2}{\Lambda^2} \right)^n = \frac{2H_0(\Lambda)}{\Lambda^2} + \frac{2H_2(\Lambda)}{\Lambda^4} (ME + \gamma_t^2) + \dots \quad (1)$$

which absorbs all dependence on the cut-off as $\Lambda \rightarrow \infty$. Eq.(1) is analytical in E and can be obtained from a three-body Lagrangian, employing a three-nucleon auxiliary field analogous to the treatment of the two-nucleon channels [14]. Contrary to the terms without derivatives, the term involves three-body forces (second term) contains two derivatives. The derivation of the integral equation describing neutron-deuteron scattering has been discussed before [15]. We present here only the result, including the new term generated by the second term in Eq.(1). The resulting amplitudes is a mixture of t_s describes the $d_t + N \rightarrow d_s + N$ process, and t_t describes the $d_t + N \rightarrow d_t + N$ process:

$$\begin{aligned}
t_s(p, k) &= \frac{1}{4} [3\mathcal{K}(p, k) + 2\mathcal{H}(E, \Lambda)] + \frac{1}{2\pi} \int_0^\Lambda dq q^2 [\mathcal{D}_s(q) [\mathcal{K}(p, q) + 2\mathcal{H}(E, \Lambda)] t_s(q) \\
&\quad + \mathcal{D}_t(q) [3\mathcal{K}(p, q) + 2\mathcal{H}(E, \Lambda)] t_t(q)] \\
t_t(p, k) &= \frac{1}{4} [\mathcal{K}(p, k) + 2\mathcal{H}(E, \Lambda)] + \frac{1}{2\pi} \int_0^\Lambda dq q^2 [\mathcal{D}_t(q) [\mathcal{K}(p, q) + 2\mathcal{H}(E, \Lambda)] t_t(q) \\
&\quad + \mathcal{D}_s(q) [3\mathcal{K}(p, q) + 2\mathcal{H}(E, \Lambda)] t_s(q)] \quad , \quad (2)
\end{aligned}$$

where $\mathcal{D}_{s,t}(q) = \mathcal{D}_{s,t}(E - \frac{q^2}{2M}, q)$ are the propagators of deuteron. For the spin-triplet S-wave channel, one replaces the two boson binding momentum γ and effective range ρ by the deuteron binding momentum $\gamma_t = 45.7025$ MeV and effective range $\rho_t = 1.764$ fm. Because there is no real bound state in the spin singlet channel of the two-nucleon system, it is better to determine the free parameters by the scattering length $a_s = 1/\gamma_s = -23.714$ fm and the effective range $r_s = 2.73$ fm at zero momentum [14, 15]. The neutron-deuteron $J = 1/2$ phase shifts δ is determined by the on-shell amplitude $t_t(k, k)$, multiplied by the wave function renormalisation

$$T(k) = Z t_t(k, k) = \frac{3\pi}{M} \frac{1}{k \cot \delta - i k} \quad . \quad (3)$$

The spine structure of the matrix elements for neutron radiative capture by deuteron is complicated, however in very low energy for this reaction we can introduced three multipole transition that is allowed by p-parity and angular momentum conservation i.e. $I^p = \frac{1}{2}^+ \rightarrow M_1$ and $I^p = \frac{3}{2}^+ \rightarrow M_1, E_2$. The parameterization of the corresponding contribution to the matrix elements and the \mathcal{M}_1 amplitude are from the magnetic moments of the nucleon and dibryon. These are well-known and will not be given here [9, 10].

The radiative capture cross section $nd \rightarrow {}^3H\gamma$ at very low energy is given by [9],

$$\sigma = \frac{2}{9} \frac{\alpha}{v_{rel}} \frac{p^3}{4M_N^2} \sum_{iLSJ} [|\tilde{\chi}_i^{LSJ}|^2] \quad (4)$$

where

$$\tilde{\chi}_i^{LSJ} = \frac{\sqrt{6\pi}}{p\mu_N} \sqrt{4\pi} \chi_i^{LSJ} \quad (5)$$

where χ is either the magnetic or electrical moment and μ_N is nuclear magneton and p is momentum of the incident neutron in the center of mass. The contribution of the electric transition E_i^{LSJ} to total cross section at energies less than 60 KeV is insignificant. Therefore, the electric quadrupole transition $E_2^{0(3/2)(3/2)}$ from the initial quartet state will not be considered at thermal energies.

We now turn to the Faddeev integral equation used in the magnetic moment calculation and also the interaction kernel included in this integral equation. Fig. 1 represents the contribution diagrams for adding photon-interaction to the Faddeev equation (2). In these diagrams photon is minimally coupled to nucleons in three-body systems. The diagrams for adding photon-interaction to the Faddeev equation up to N²LO are depicted in Fig. 2. Photon is coupled to two-body system via L_1 vertices.

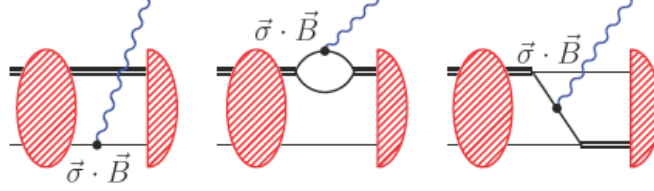


FIG. 1: (Color online) Some diagrams for adding photon-interaction to the Faddeev equation up to $N^2\text{LO}$. Thick solid line is propagator of the two intermediate auxiliary fields D_s and D_t , denoted by \mathcal{D} ; \mathcal{K} : propagator of the exchanged nucleon. Photon is minimally coupled to nucleons in three-body systems. Wavy line shows photon and small circles show magnetic photon interaction.

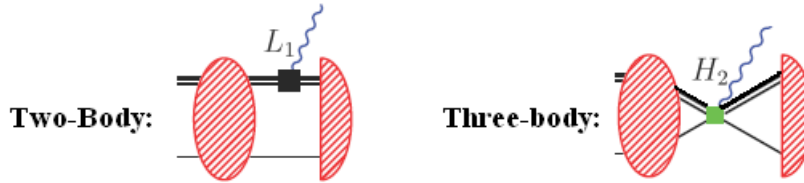


FIG. 2: (Color online) Some diagrams for adding photon-interaction to the Faddeev equation up to $N^2\text{LO}$. Photon is minimally coupled to two-body system and three-body vertices in three-body systems. For L_1 vertices, see Ref [16]; H_2 :three- body force, see eq.(1). Remaining notation as in Fig. 1.

The coefficient L_1 is fixed at its leading non-vanishing order by the thermal cross section [16]. We have other possible diagram that can be considered for our calculation for inclusion of photon to the three-body vertices \mathcal{H} . This diagram is shown in Fig. 2.

All corrections contribute to observables typically as $Q^n = \left(\frac{p_{\text{typ}}}{\Lambda_\pi}\right)^n$ compared to the LO result and that low-energy observables must be independent of an arbitrary regulator Λ up to the order of the expansion. In other words, the physical scattering amplitude must be dominated by integrations over off-shell momenta q in the region in which the EFT is applicable, $q \lesssim \Lambda_\pi$. Typical low momentum scales p_{typ} in the three-body system are the binding momenta of the two-nucleon real and virtual bound states, $\gamma_s \approx -8.0$ MeV, $\gamma_t \approx 45$ MeV and the scattering momentum k . In addition, the three-body forces are determined in part by the typical three-nucleon bound state momentum $\gamma_d \sim \sqrt{MB_d} \approx 90$ MeV, B_d is the triton binding energy. The breakdown scale $\Lambda_\pi \approx m_\pi$ of the theory is the scale at which higher order corrections become comparable in size. One can therefore estimate sensitivity to short-distance physics, and hence provide a reasonable error analysis, by employing a momentum cut-off Λ in the solution of the Faddeev equation and varying it between the breakdown-scale Λ_π to ∞ . If observables change over this range by “considerably” more than Q^{n+1} , a counter-term of order Q^n should be added. This method is frequently used to check the power counting and systematic errors in pionless EFT with three nucleons, see e.g. most recently [15]. A similar argument was also developed in the context of the EFT “with pions” in nuclear physics [17, 18].

TABLE I: Comparison between different theoretical and experimental results for Neutron radiative capture by deuteron at zero energy (0.0253 eV). Last row shows N²LO order pionless EFT result.

Experiment	Year	Total cross section(mb)
Jurney et.al. [11]	1963	0.60 ± 0.05
Merritt et.al. [12]	1968	0.521 ± 0.009
Jurney et.al. [13]	1982	0.508 ± 0.015
Theory		
AV14/VIII(IA+MI+MD+ Δ) [7]	1996	0.600
AV18/IX(IA+MI+MD+ Δ) [7]	1996	0.578
AV18/IX (gauge inv.) [8]	2005	0.523
AV18/IX (gauge inv. + 3N-current) [8]	2005	0.556
EFT(N ² LO)+3N-forces [10]	2006	0.503 ± 0.003

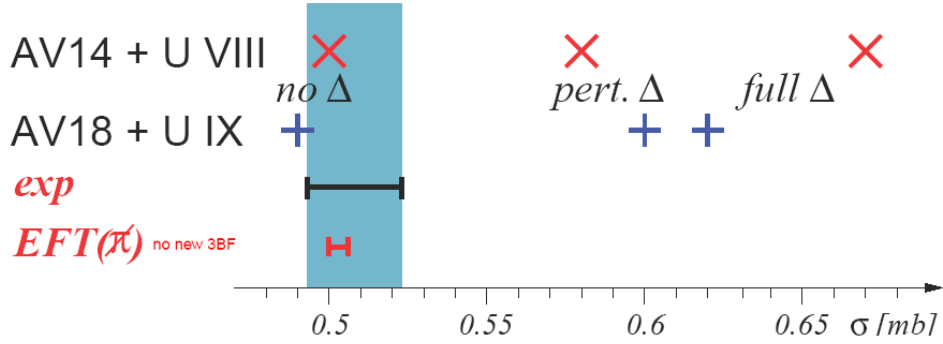


FIG. 3: (Color online) Comparison between calculated cross section of neutron radiative capture by deuteron by different theoretical models, pionless EFT and experimental data.

III. PHOTON POLARIZATION PARAMETER AT THERMAL ENERGIES

If the process is dominated by S-wave capture, as in the case for the neutron-deuteron radiative capture reaction at thermal neutron energies, the observable for circular polarization $P_{\Gamma}(\theta)$ is simply given by:

$$P_{\Gamma}(\theta) = R_c P_N \cdot \hat{q} \quad (6)$$

where P_N is the polarizations of the spin-1/2 nucleon and R_c is the polarization parameter (for more detail see [7]). This polarization parameter depends on the relative sign between the amplitudes 1/2 and 3/2 channels. Numerically, R_c lies in the region $-1/2 \leq R_c \leq 1$. Its experimental value is $R_c = -0.42$ [1].

IV. RESULTS

We numerically solved the Faddeev integral equation up to N^2LO . We used $\hbar c = 197.327$ MeV fm, and mass of $M = 938.918$ MeV for nucleon. A deuteron binding energy (momentum) of $B = 2.225$ MeV ($\gamma_d = 45.7066$ MeV) is used for the nucleon-nucleon triplet channel. A residue of $Z_d = 1.690(3)$ is used for the NN singlet channel. The 1S_0 scattering length is chosen to be $a_s = -23.714$ fm. After fixing the leading non-vanishing order in the thermal cross section L_1 is found to be -4.5 fm.

As in Ref. [10], we determined which three-body forces are required at any given order, and how they depend on the cut-off. Low energy observables must be insensitive to the cut-off, namely to any details of short range physics in the region above the break down scale of the pionless EFT (which set approximately by the pion mass).

The results for the thermal energy cross section and photon polarization parameter are presented in table I and II, along with the experimental data [1, 13]. Table I compares the $nd \rightarrow ^3H\gamma$ cross section at zero energy (0.0253 eV) for various experimental and theoretical works. The corresponding values for the cross section from the pionless EFT evaluation up to N^2LO is shown in the last row. The EFT results for this cross section are presented only up to three significant digits.

Recently in a model dependent two-body current calculation, the total cross section for $nd \rightarrow ^3H\gamma$ is obtained as $\sigma_T = 0.523$ mb [8]. This value can be compared with the corresponding result $\sigma_T = 0.558$ mb obtained in Ref. [7]. The later work used the present MI two-body current operators therefore leads to an estimate closer to the experimental data $\sigma_T = 0.508 \pm 0.15$ mb [13]. However, the addition of the three-body currents, which give a rather sizable contribution as can be seen from the row labeled “full-new” in table I, brings the total cross section to $\sigma_T = 0.556$ mb. Table I shows also EFT result of the cross section for this reaction up to N^2LO order. Three-nucleon forces are needed up to N^2LO order for cut-off independent results. Hence the cross-section is in total determined as $\sigma_{tot} = [0.485(LO) + 0.011(NLO) + 0.007(N^2LO)] = [0.503 \pm 0.003]mb$. The theoretical accuracy may for example be estimated conservatively by $Q \sim \frac{\gamma_t}{m_\pi} \approx \frac{1}{3}$ of the difference between the NLO and N^2LO results.

Table II shows Comparison between the results of different models dependent, model independent EFT and experiment for the photon polarization parameter. The photon polarization parameter is sensitive to two-body currents (for its definition in terms of RMEs, see Ref. [7]). We compare our prediction for the photon polarization parameter with the theoretical and the experimental results of Ref. [1, 7, 8] in this table. The magnetic M_1 -transition gives the dominant contribution for our calculation.

In Fig. 3, we compare our results with those obtained in Refs. [7, 13]. As can be seen by inspecting Fig. 3, the pionless EFT calculations is converges order by order in low energy expansion and also cut-off independent at this order(see [10]). There are no new three-nucleon forces besides those already fixed in nd scattering at the same order. The contribution from the photon coupling to a three-nucleon force is negligible in our calculation. Our calculation has a systematic error which is now smaller than the experimental error-bar.

V. SUMMERY AND CONCLUSION

We applied pionless EFT to find numerical results for the photon polarization parameter R_C . At very low energies, the interactions between nucleons can be described only by point-like interactions.

TABLE II: Comparison between different theoretical results for photon polarization parameter R_C of the reaction neutron-deuteron radiative capture at thermal energies. The last line quotes deviation between data and theory, if it is larger than the theoretical or experimental uncertainty.

Type of theory and experiment	R_C	Overestimation the experimental
AV14/VIII(IA+MI+MD+ Δ) [7]	-0.420	below 1%
AV18/IX(IA+MI+MD+ Δ) [7]	-0.469	12%
AV18/IX (gauge inv.+ 2N-current) [8]	-0.429	2%
AV18/IX (gauge inv. + 3N-current) [8]	-0.476	13%
EFT(LO)	-0.387	8%
EFT(NLO)	-0.403	4%
EFT(N ² LO)	-0.412	2%
Experiment [1]	-0.42 ± 0.03	

One cannot identify pions as the lightest exchanged particles between nucleons as long as the typical external momentum p_{typ} in a reaction is below the pion mass m_π . That is because the Compton wavelengths are not small enough to resolve the nuclear forces as originating in part from one pion exchange. Then all particles but nucleons are integrated out. One can identify a small, dimensionless parameter $Q = \frac{p_{\text{typ}}}{\Lambda_\pi} \ll 1$, where $\Lambda_\pi \sim m_\pi$ is the typical momentum scale at which the one pion exchange is resolved and pionless EFT must break down. Incident thermal neutron energies have been considered for this capture process.

The photon polarization parameter R_c of the reaction neutron-deuteron radiative capture $nd \rightarrow \gamma^3H$ at thermal energies was calculated in pionless EFT. This model independent and systematic low energy version of QCD is suited for processes involving momenta below the pion mass. At these energies our calculation is dominated by S -wave state and magnetic transition M_1 contribution only. The M_1 amplitude is calculated up to N²LO. Three-nucleon forces are needed up to N²LO order for cut-off independent results. The triton binding energy and nd scattering length in the triton channel have been used to fix them. Hence the The photon polarization parameter in total is determined as $R_c = -[0.387(LO) + 0.016(NLO) + 0.009(N^2LO)] = [-0.412 \pm 0.003]$. This converges order by order in low energy expansion and also is cut-off independent at this order. We notice that our calculation has a systematic error which is now smaller than the experimental error bar.

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